Name: ______________________________

Instructor: __________________________

Math 10120, Exam I
September 16, 2006

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

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Multiple Choice	______
11.	______
12.	______
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Total	______

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Multiple Choice

1.(5 pts.) Consider the following sets:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$
$$A = \{2, 4, 6, 8, 10\}$$
$$B = \{1, 2, 3, 4, 5\}.$$

Then $A \cap B'$ is the set

We have

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$
$$B' = \{\cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, 6, 7, 8, 9, 10\} = \{6, 7, 8, 9, 10\}$$
$$A = \{2, 4, 6, 8, 10\}.$$

The elements shown in red are common to both sets, hence

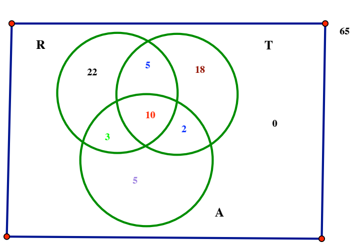
$$A \cap B' = \{6, 8, 10\}.$$

(a) $\emptyset$ (b) $\{1, 3, 5, 6, 7, 8, 9, 10\}$ (c) $\{6, 8, 10\}$

(d) $\{7, 9\}$ (e) U

2.(5 pts.) A survey of 65 students revealed that 40 of them liked Rap, 20 of them liked Alternative Music, 15 of them liked Rap and Techno, 12 of them liked Techno and Alternative music, 5 of them like Alternative music only and 10 of them like all three types of music. Assume that <u>every student likes at least one of the music types</u>. The number of students who like Techno is

U	65
R	40
A	20
$R \cap T$	15
$T \cap A$	12
$T' \cap R' \cap A$ (A only)	5
$T \cap R \cap A$	10
$(R \cup T \cup A)'$	0



The numbers shown in the diagram result from putting the numbers from the table on the left into the diagram in sequence, first black, then red, then purple, then blue, then green. Finally when we add the numbers in the diagram subtract from 65 we get that the number in T only is 18 and the number who like Techno is $n(T) = 18 + 5 + 10 + 2 = 35$.

(a) 50 (b) 30 (c) 23

(d) 35 (e) none of the above

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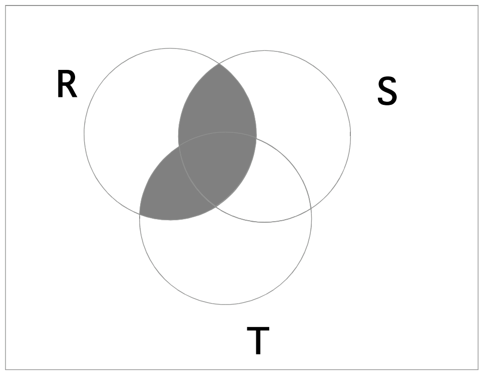
3.(5 pts.) Which of the following sets is represented by the shaded region in the Venn diagram below?



(a) $R \cap T$ does not contain anything outside R and T, Since part of the shaded region above is outside of R and T, we can eliminate (a).
(c) and (e) Since $R \cup (S \cap T)$ and $R \cup S \cup T$ include everything in R, and since the shaded region above is missing part of R, we can eliminate answers (c) and (e).
(c) $R \cap (S \cup T)'$; This is the set of all elements that inside R and outside $S \cup T$. These elements are in R only and are not in the shaded region above. Therefore we can eliminate (c).
Now quoting Sherlock Holmes:
"when you have eliminated the impossible, whatever remains, however improbable, must be the truth."
Therefore the answer must be (b).

You can reason your way to this conclusion in many ways. You can use verbal descriptions of sets to check if the shaded region given matches, or you can draw a sketch of the regions described by answers (a)-(e) and check which one matches the diagram given.

(a) $R \cap T$ (b) $R \cap (S \cup T)$ (c) $R \cup (S \cap T)$

(d) $R \cap (S \cup T)'$ (e) $R \cup S \cup T$

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4.(5 pts.) Consider the following sets:

$$U = \{\text{All students at Notre Dame}\}$$

$$A = \{\text{All female students at Notre Dame}\}$$

$$B = \{\text{All Students under 20 years of age at Notre Dame}\}$$

Then $(A \cup B)'$ is the set (Hint: you may want to use one of DeMorgan's Laws)

By DeMorgan's Laws, we have $(A \cup B)' = A' \cap B'$.
This is the set of people in the universal set who are NOT in A AND NOT in B.
These are the students at Notre Dame who are NOT Female AND who are NOT under 20 years of age,
in other words they are the male students at Notre Dame who are at least 20 years of age.

(a) All students at ND who are male OR at least 20 years of age

(b) All male students at ND who are at least 20 years of age

(c) All male students at ND who are under 20 years of age

(d) All female students at ND who are under 20 years of age

(e) All male students at ND

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5.(5 pts.) In how many ways can First, Second and third prize be awarded in an art contest with 14 entries, if no prize can be shared?

We break the process of awarding prizes into 3 steps:
Step 1: Award 1st prize (14 ways)
Step 2: Award 2st prize (13 ways)
Step 3: Award 3st prize (12 ways)

In all there are $14 \cdot 13 \cdot 12 = P(14, 3)$ ways to award the prizes.

(a) 14^3 (b) $C(14, 3)$ (c) $P(14, 3)$ (d) $14!$ (e) 3^{14}

6.(5 pts.) How many four-letter words (including nonsense words) can be made from the letters of the word

IRRITAINMENT

assuming that letters cannot be repeated.

We must first eliminate repeated letters:

$$I\ R\ \cancel{R}\ \cancel{I}\ T\ A\ \cancel{I}\ N\ M\ E\ \cancel{N}\ \cancel{T}$$

This leaves us with 7 letters with which to make four letter words with no repeating letters. Again we break the task of creating a four letter word into steps:
Step 1: Choose the first letter (7 ways)
Step 2: Choose the second letter (only 6 ways because I cannot repeat the first letter)
Step 3: Choose a third letter (5 ways)
Step 4: Choose the fourth letter (4 ways)
In all I can make $7 \cdot 6 \cdot 5 \cdot 4$ such words.

(a) 7^4 (b) 12^4 (c) $\dfrac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1}$

(d) $\dfrac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1}$ (e) $7 \cdot 6 \cdot 5 \cdot 4$

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7.(5 pts.) Coach Brown has 16 Volleyball players on her squad. She wants to choose 6 to go on court at the beginning of tonight's game and 4 substitutes. In how many ways can she do this?

We can break the task up into 2 steps:

Step 1: Choose 6 players to go on court: $C(16, 6)$ ways.

Step 1: Choose 4 of the remaining players as substitutes: $C(10, 4)$ ways.

In all there are $C(16, 6) \cdot C(10, 4)$ ways of completing this task.

(a) $C(16, 10)$ (b) $\frac{C(16,6) \cdot C(10,4)}{2 \cdot 1}$ (c) $C(16, 6) \cdot C(10, 4)$

(d) $P(16, 6)P(10, 4)$ (e) $16!$

8.(5 pts.) Jason is about to compete in a race in the An Tostal Festival. He can hop or roll from the start of the race to point A, he can skip, run sideways or run backwards from point A to Point B and he can crawl, or walk on his heels from point B to the finish line. In how many ways can Jason complete the course?

Breaking the race into steps

Step 1: Go from the START to POINT A (2 ways)

Step 2: Go from POINT A to POINT B (3 ways)

Step 3: Go from POINT B to FINISH LINE (2 ways)

In all there are $2 \cdot 3 \cdot 2 = 12$ ways to complete the course.

(a) 12 (b) 7 (c) 3 (d) 14 (e) 5

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9.(5 pts.) Charles has 1000 songs on his ipod. He is about to go jogging and wants to choose an ordered list of 15 songs to listen to while he is jogging. How many possible lists of 15 songs can he make from the 1000 songs on his ipod.

Here Charles is choosing 15 songs from 1000 and the order in which he chooses matters, hence he is choosing a permutation of 15 songs from 1000. There are

$$P(1000, 15)$$

such permutations.

(a) $P(1000, 15)$ (b) $1000!$ (c) $15!$

(d) 1000 (e) 1000^{15}

10.(5 pts.) Suppose an experiment consists of tossing a coin 5 times and observing the sequence of heads and tails. How many different outcomes have at least one head?

The complement of the event "At Least ONe Head" is the event "No Heads"
The number of sequences with at least one head =
Total Number of Sequences - Number of sequences with no heads =

$$2^5 - C(5, 0).$$

(a) 2^5 (b) 5^2 (c) $C(5, 1)$

(d) $C(5, 0) + C(5, 1)$ (e) $2^5 - C(5, 0)$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(10 pts.) (a) Let S and T be sets. Give an eqaution relating $n(S \cup T)$, $n(S)$, $n(T)$ and $n(S \cap T)$.

The inclusion-exclusion principle says that

$$n(S \cup T) = n(S) + n(T) - n(S \cap T).$$

(b) If A is a subset of a universal set U, give an equation relating $n(A)$, $n(A')$ and $n(U)$.

The complement rule says:

$$n(A) + n(A') = n(U).$$

(c) If a set, X, has 10 elements, how many subsets does X have?

The number of subsets of size R is the number of ways of choosing a sample R objects from the 10 which is $C(10, R)$.

$$C(10, 0) + C(10, 1) + C(10, 2) + \cdots + C(10, 10) = 2^{10}.$$

Recall that this is equivalent to adding the line of Pascal's triangle with $n = 10$.

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12.(10 pts.) Recall that a poker hand consists of a sample of 5 cards drawn from a deck of 52 cards. The deck has 13 spades, 13 clubs, 13 diamonds and 13 hearts.

(a) How many poker hands have 3 diamonds and 2 spades?

We imagine choosing such a hand in steps:
Step 1: Choose 3 diamonds : $C(13, 3)$ ways
Step 2: Choose 2 spades : $C(13, 2)$ ways
Therefore the number of hands with 3 diamonds and 2 spades is

$$C(13, 3) \cdot C(13, 2) = 22,308.$$

(b) Recall also that a deck of 52 cards can be divided into 13 denominations,(two's through aces), with 4 cards in each denomination. How many poker hands have 3 cards from one denomination and 2 from another? (a house)

Again, we imagine choosing such a hand in steps:
Step 1: Choose the first denomination, 13 ways
Step 2: Choose 3 cards from that denomination, $C(4, 3)$ ways
Step 3: Choose the second denomination, 12 ways
Step 4: Choose 2 cards from that denomination, $C(4, 2)$ ways
Therefore the number of hands with 3 cards from one denomination and 2 from another is

$$13 \cdot C(4, 3) \cdot 12 \cdot C(4, 2) = 3,744$$

(c) How many poker hands are there with 4 Aces?

Again, we imagine choosing such a hand in steps:
Step 1: Choose 4 aces, 1 way (since there are only 4 aces available)
Step 2: Choose another card, 48 ways

Therefore the number of hands with 4 aces is 48

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13.(10 pts.) The Notre Dame Squash club has 15 members.

(a) If the club wishes to organize a round robin for its members, (every player plays every other player exactly once), how many matches must be played?

The number of matches = the number of ways we can draw a pair of players from the 15 members

$$= C(15, 2) = 105 \text{ matches.}$$

(b) In a survey of the 15 club members it was found that 10 liked to play with Dunlop racquets, 6 liked to play with Black Knight racquets and 2 didn't like to play with either brand. How many of the club members liked to play with both Black Knight and Dunlop racquets?

Let B denote the set of members who like to play with Black Knight racquets and let D denote the set of members who like to play with Dunlop racquets.

The information we are given can be summarized as:

$$n(D) = 10,\ n(B) = 6,\ n((B \cup D)') = 2,\ n(U) = 15.$$

We know that

$$n(B \cup D) = n(B) + n(D) - n(B \cap D).$$

Filling in what we know, we get

$$n(B \cup D) = 6 + 10 - n(B \cap D).$$

Using the complement rule, we get

$$n(B \cup D) = n(U) - n((B \cup D)') = 15 - 2 = 13.$$

Putting this into the above equation, we get

$$13 = 16 - n(B \cap D)$$

or

$$n(B \cap D) = 3.$$

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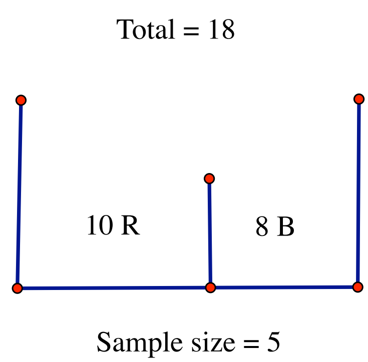
14.(10 pts.) An urn contains 10 numbered red balls and 8 numbered blue balls. A random sample of size 5 is drawn from the urn.

We first draw a picture of the urn



(a) How many such samples can be drawn?

Since there are a total of 18 balls in the urn, we can draw $C(18,5) = 8568$ different samples of size 5.

(b) How many of the above samples consist of 2 red balls and 3 blue balls?

We break the task of drawing such a sample into two steps;
Step 1: Draw 2 red balls from the urn, $C(10,2)$ ways,
Step 2: Draw 3 blue balls from the urn, $C(8,3)$ ways,
The number of ways to draw such a sample is

$$C(10,2)\cdot C(8,3) = 2520.$$

(c) How many samples of size five have at least one red ball in them?

Here we use the complement rule:
The complement of the set of samples of size 5 with "At least one Red" is the set of samples of size 5 with "No Red" = "All Blue".
Therefore the number of samples with at least one red is
Total number of samples (of size 5) - number of samples (of size 5) with 5 Blue balls

$$= C(18,5) - C(8,5) = 8512.$$

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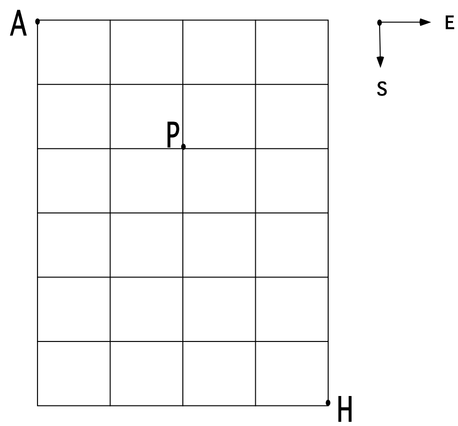
15.(10 pts.) On arriving at LAX airport (at A), in El Pueblo de Nuestra Senora la Reina de losAngeles de Porciuncula, Arnold takes a cab to his home (at H). The grid shown below gives a street map of the area.



(a) If the cab driver always travels south or east (no backtracking), how many routes can he take from the airport at A to Arnold's home at H?

The cab driver must travel 10 blocks with exactly 4 in an easterly direction. There are $C(10,4) = 210$ such routes.

(b) How many of those routes pass through the intersection at P?

The number of such routes from A to H which pass through P is
(The number of such routes from A to P) $\times$ (The number of such routes from P to H.)

$$= C(4,2) \times C(6,2) = 6 \times 15 = 90.$$

(c) If the cab driver knows that there are roadworks at P, how many routes can he take that do not pass through P?

The number of such routes from A to H which DO NOT pass through P is
(The total number of such routes from A to H) - (The number of such routes from A to H which pass through P)

$$= C(10,4) - C(4,2) \times C(6,2) = 210 - 90 = 120.$$

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Multiple Choice	________
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